DISTANCE AND MIDPOINT FORMULAS

EXAMPLE: Find and simplify the distance between the following sets of points:

1. With $(-2, 3) = (x_0, y_0)$ and $(1, -3) = (x_1, y_1)$, we get

$$\begin{aligned} d &= \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} \\ &= \sqrt{(1 - (-2))^2 + (-3 - 3)^2} \\ &= \sqrt{9 + 36} \\ &= \sqrt{45} \\ &= \sqrt{9 \cdot 5} \\ &= \sqrt{9}\sqrt{5} && \text{For nonnegative numbers, } \sqrt{ab} = \sqrt{a}\sqrt{b}. \\ &= 3\sqrt{5} \end{aligned}$$

2. With $\left(\frac{1}{2}, \frac{2}{3}\right) = (x_0, y_0)$ and $\left(\frac{3}{4}, \frac{1}{5}\right) = (x_1, y_1)$, we get

$$\begin{aligned} d &= \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} \\ &= \sqrt{\left(\tfrac{3}{4} - \tfrac{1}{2}\right)^2 + \left(\tfrac{1}{5} - \tfrac{2}{3}\right)^2} && \text{Get common denominators to add and subtract fractions.} \\ &= \sqrt{\left(\tfrac{1}{4}\right)^2 + \left(-\tfrac{7}{15}\right)^2} \\ &= \sqrt{\tfrac{1}{16} + \tfrac{49}{225}} && \text{Since } \left(\tfrac{a}{b}\right)^2 = \tfrac{a^2}{b^2},\ b \neq 0. \\ &= \sqrt{\tfrac{1009}{3600}} \\ &= \tfrac{\sqrt{1009}}{\sqrt{3600}} \\ &= \tfrac{\sqrt{1009}}{60} && \text{For nonnegative numbers, } \sqrt{\tfrac{a}{b}} = \tfrac{\sqrt{a}}{\sqrt{b}},\ b \neq 0. \end{aligned}$$

EXAMPLE: Find the midpoint of the line segment connecting the following pairs of points:

1. With $(-2, 3) = (x_0, y_0)$ and $(1, -3) = (x_1, y_1)$, we get

$$\begin{aligned} M &= \left(\frac{x_0 + x_1}{2}, \frac{y_0 + y_1}{2}\right) \\ &= \left(\frac{(-2) + 1}{2}, \frac{3 + (-3)}{2}\right) = \left(-\frac{1}{2}, \frac{0}{2}\right) \\ &= \left(-\frac{1}{2}, 0\right) \end{aligned}$$

   The midpoint is $\left(-\frac{1}{2}, 0\right)$.

2. With $\left(\frac{1}{2}, \frac{2}{3}\right) = (x_0, y_0)$ and $\left(\frac{3}{4}, \frac{1}{5}\right) = (x_1, y_1)$, we get

$$\begin{aligned} M &= \left(\frac{x_0 + x_1}{2}, \frac{y_0 + y_1}{2}\right) \\ &= \left(\frac{\frac{1}{2} + \frac{3}{4}}{2}, \frac{\frac{2}{3} + \frac{1}{5}}{2}\right) \\ &= \left(\frac{\left(\frac{1}{2} + \frac{3}{4}\right) \cdot 4}{2 \cdot 4}, \frac{\left(\frac{2}{3} + \frac{1}{5}\right) \cdot 15}{2 \cdot 15}\right) \quad \text{Simplify compound fractions.} \\ &= \left(\frac{5}{8}, \frac{13}{30}\right) \end{aligned}$$

   The midpoint is $\left(\frac{5}{8}, \frac{13}{30}\right)$.